A FULL-WAVE ANALYSIS FOR MICROWAVE, PLANAR, DISTRIBUTED DISCONTINUTITES

Barry E. Spielman and Louis G. Chen*
Washington University
Department of Electrical Engineering
St. Louis, MO 63130

* Was with the Department of Electrical Engineering,
Washington University in St. Louis.
Now with AT&T Bell Laboratories, Andover, MA

Abstract:

A full-wave analysis is presented for modeling microwave, planar, distributed discontinuities. By choosing current-density basis functions that better match expected singular behavior of the current density near conductor edges , implementing a mode-matching method, and approximating the distributed discontinuity by a multi-step structure, this method is found to be more efficient for studying the characterisitics of the distibuted discontinuity than similar, previously-reported methods. A combination of scattering matrices is used to numerically approximate the behavior of the distributed discontinuity. Simulation results are given for some specific illustrations, which exhibit good agreement with other known work.

I ANALYSIS METHOD

This paper presents an adaptable, full-wave analysis for accurately determining the performance of planar, distributed discontinuities at higher microwave and millimeter-wave frequencies. The salient mathematical features of this analysis are presented. The distributed discontinuity is represented as a sequence of uniform microstrip lines of different width joined by abrupt, step-like discontinuities, as illustrated in Fig. 1 for a linear microstrip taper. The analysis

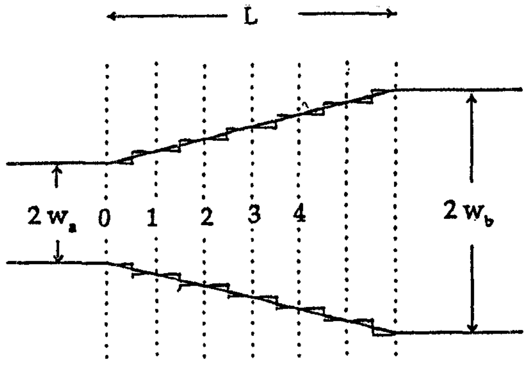


Fig.1. Representation of a tapered microstrip using multi-step model

is comprised of three aspects: (1) the full-wave modeling of dispersion characteristics on the uniform lines between step discontinuities, (2) the full-wave modeling of each step discontinuity, and (3) the appropriate cascading of these models to represent the behavior of the distributed discontinuity in terms of S parameters. This paper presents the essential features of these three aspects.

For aspect (1), the spectral domain method is used to transform field variables and express the longitudinal (z-directed) and transverse (x-directed) electric and magnetic fields along the plane of the dielectric-to-air interface shown in Fig. 2 in terms of longitudinal and transverse current-

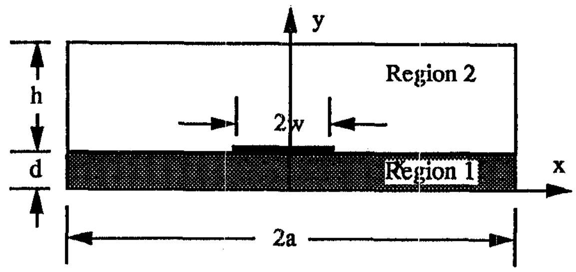


Fig.2. The cross section of a uniform microstrip

density distributions flowing in conductors at that interface. These current-density distributions are expanded using Legendre-polynomial basis functions for the longitudinal current densities and harmonic functions for the transverse densities. Legendre polynomials are selected because these functions more accurately represent the actual current-density singularity experienced in longitudinal currents at conductor edges, even when few basis functions are used.

A Galerkin solution is used to invoke the condition that the tangential E and H fields are continuous across the dielectric-to-air boundaries in Fig. 2 and that the tangential E field is zero at conductors along the same plane. In this fashion the problem is cast into the matrix form [C] [AB] = 0 = 1 [AB]. Elements of [C] depend on geometry and material parameters for the particular microstrip configuration. [AB] is an array of coefficients which weight the selected Legendre and harmonic current-density basis functions. The phase coefficient b is determined for zeros in the determinant of [C] as a function of frequency, both for propagating and evanescent modes. These dispersion characteristics are also used in the modeling of step discontinuities.

To understand the treatment used for the step discontinuity, consider Fig. 3. The full-wave analysis for treating such

WE 3F

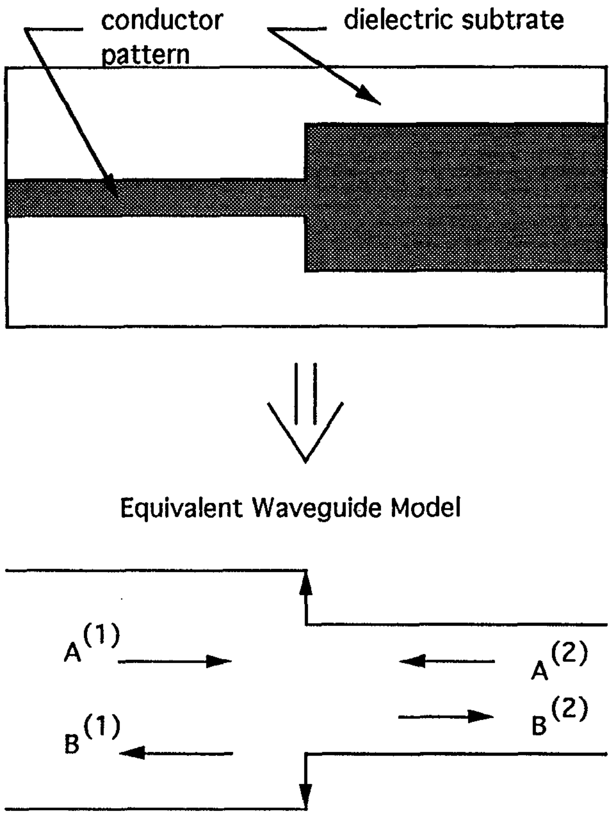


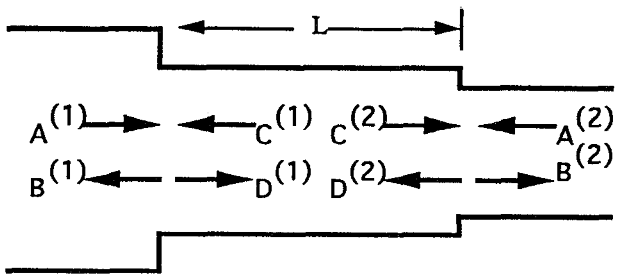
Fig. 3. An equivalent waveguide model of a microstrip with an abrupt junction

discontinuities uses boundary reduction [2], wherein more accurate and efficient solutions are obtained by assuming that the incident signal approaches the step from the side with the narrower microstrip line. The behavior of the input and output field distributions for each step are reduced to matrix form using a moment solution that uses modal magnetic fields to test transverse electric field expansions at the junction and modal electric fields to test transverse magnetic field expansions at the junction. The modal fields are obtained from the solution for the uniform line. A sufficient number of modes must be used to obtain the desired accuracy and efficiency. The resulting matrix representation is then reorganized into a generalized scattering matrix representation to relate input and output field distributions at each step junction.

The behavior of the overall distributed junction is then represented as a sequence of interconnected uniform lines and step discontinuities. This mathematical treatment is illustrated by developing the S-parameter representation for the pair of microstrip step junctions interconnected by a length of uniform microstrip line of length l as depicted in Fig. 4.

II SIMULATIONS AND NUMERICAL RESULTS

A linear, tapered microstrip discontinuity was selected to illustrate the effectiveness of the approach described here. The results presented in the subsequent sections are intended to benchmark the accuracy and efficiency of each aspect of the analysis.



$$B^{(1)} = S_{AA}\,A^{(1)} + S_{AB}\,A^{(2)}$$

$$B^{(2)} = S_{BA}\,A^{(1)} + S_{BB}\,A^{(2)}$$

Fig. 4. The scattering matrix formulation for two junctions

A. UNIFORM, SHIELDED MICROSTRIP LINE

To benchmark the treatment of dispersion characteristics for uniform lines, consider the microstrip cross section shown in Fig. 5. Even-degree Legendre polynomials were used for the longitudinal current basis functions and sin(ipx) (i = 1, 2, ...) for the transverse current basis functions. Fig. 5 shows calculated values of phase coefficient versus frequency for the fundamental mode and three higher-order modes compared with well-documented results of Ganguly and Spielman [3]. Ganguly and Spielman employed a full-wave, method-of-moments, boundary-element (pulse expansion functions with point matching) solution. The results evaluated here employed three Legendre basis functions and three harmonic functions. The results produced here demonstrate improved computational efficiency compared with earlier work as described in the discussion section of this paper. Fig. 6 shows convergence results for the

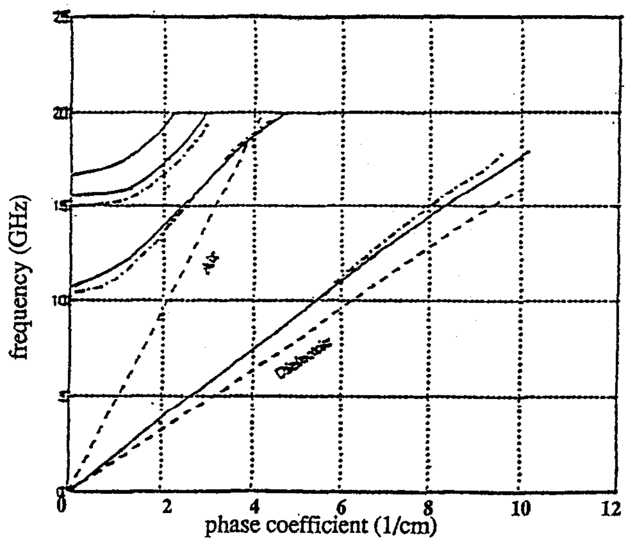


Fig.5. Dispersion characteristics of eigenmodes in a uniform microstrip (w=0.635mm, a=6.35mm, h=11.43mm, s=1.27mm, $\varepsilon_r = 8.875$) ———— by Chen&Spielman, _ . _ . _ by Ganguly&Spielman [3]

longitudinal current density as the number of Legendre basis functions is increased. Fig. 7 shows longitudinal current density for the fundamental and first higher-order mode computed using only five Legendre basis functions.

B. DISTRIBUTED DISCONTINUITY - TAPERED MICROSTRIP JUNCTION

The computed performance for the tapered microstrip line depicted in Fig. 1 is dependent upon the number of uniform microstrip line sections used to represent the tapes, the number of harmonic terms used to represent the transverse current density on the uniform lines, the finite number of modes (some propagating, some evanescent) employed to represent behavior between step junctions, and the number of Legendre basis functions used to represent the longitudinal current density on the uniform lines. Figs. 8 -11 show convergence results for S_{11} (or VSWR) as each of these quantities is varied, respectively. Fig. 12 show simulation results of the magnitude of S_{11} versus frequency for a linearly tapered microstrip compared with corresponding results of Rao and Kosta [4]. Information will also be presented about the phase behavior of S_{11} for this simulation.

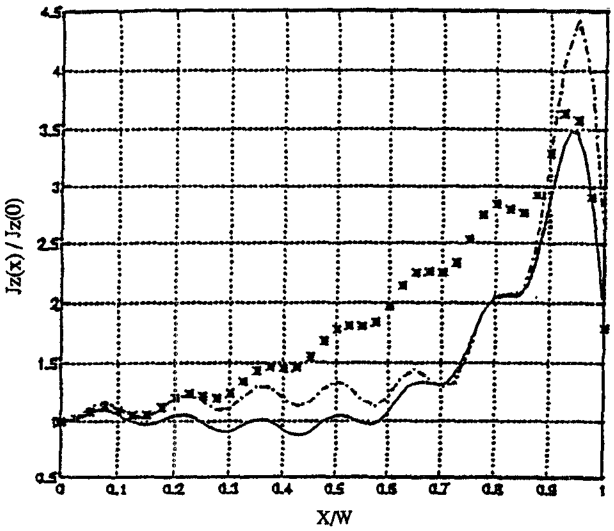


Fig. 6 Normalized longitudinal current distributions for fundamental mode vs the number of basis functions(w=0.635mm, a=6.35mm, h=12.7 mm, d=1.27mm, ε_r =10, f=8GHz)

*****number of Legendre basis functions=2——number of Legendre basis functions=3, -·-·- number of Legendre basis functions=4

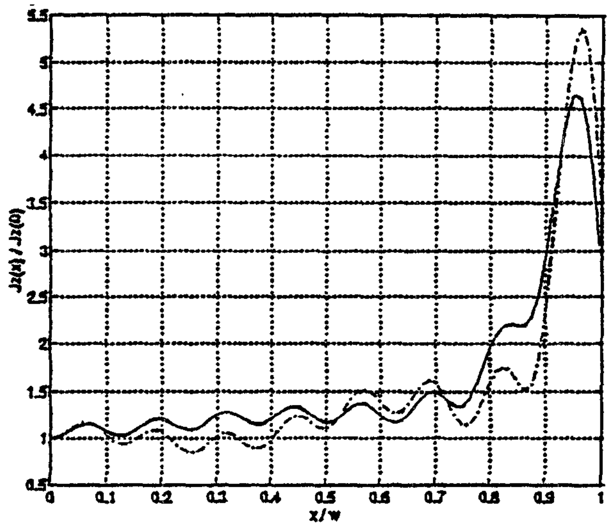


Fig. 7. Normalized longitudinal current distributions for fundamental mode and the first higher order mode(w=0.635mm. a=6.35mm, h=12.7mm, d=1.27mm, ε_r=10, f=12GHz, harmonic terms=160, the number of Legendre basis functions=5)——— the fundamental modes, -·-·- the first higher order mode.

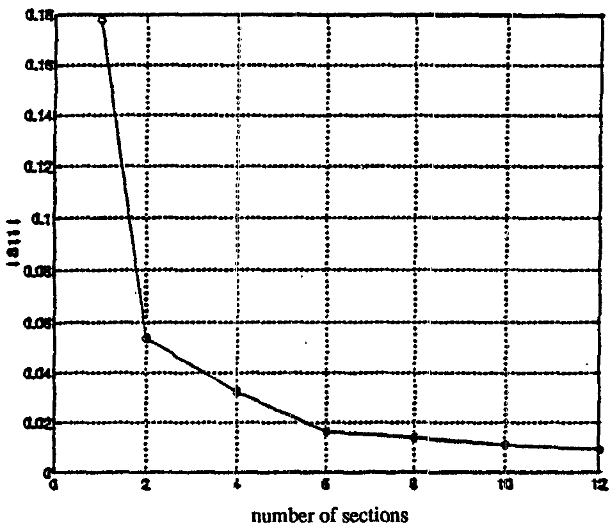


Fig. 8. Convergence test of $|S_{11}|$ vs the number of sections (w_a=0.25mm, w_b=0.635mm, a=6.35mm, h=12.7mm, d=1.27mm, L=20mm, ε_r=10, f=4.5GHz, harmonic terms=140, the number of modes=2, the number of Legendre basis functions=3) for a microstrip taper

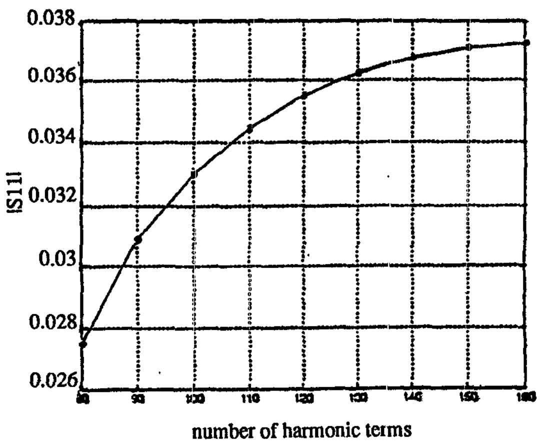


Fig. 9. Convergence test of $|S_{11}|$ vs the number of harmonic terms (w_a=0.25mm, w_b=0.635mm, a=6.35mm, h=12.7mm, d=1.27mm, L=20mm, ε_r=10, f=4.5GHz, number of sections=8, the number of modes=2, the number of Legendre basis functions=3)

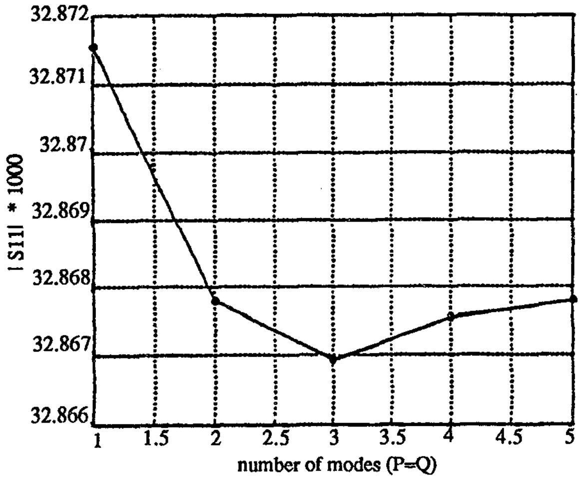


Fig. 10. Convergence test of | S_{11} | vs the number of modes (w_a=0.25mm, w_b=0.635mm, a=6.35mm, h=12.7mm, d=1.27mm, L=20mm, ε_r=10, f=4.5GHz, number of sections=8, the harmonic terms=140, the number of basis functions=3)

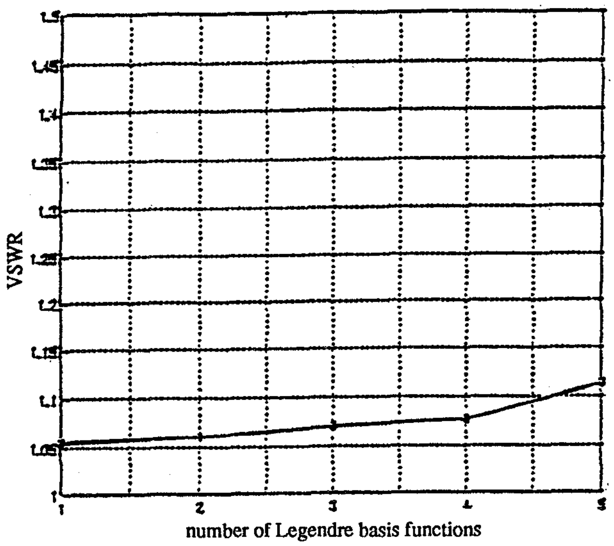


Fig. 11. Convergence test of VSWR vs the number of Legendre basis functions (w_a=0.25mm, w_b=0.635mm, a=6.35mm, h=12.7mm, d=1.27mm, L=20mm, ε_r=10, f=4.5GHz, number of sections=8, the harmonic terms=140, the number of modes=2)

III. DISCUSSION

The accuracy and efficiency of each of the three aspects of the analysis must be considered. The roots of the determinant of the [C] matrix described in section I exhibit distinct values for true characteristic modes. The accuracy of computed values for these modes is typically within about 5% compared with well-documented results determined using other methods [3]. It is noteworthy that it takes just one-fifth the computation time to generate dispersion characteristics using the approach described here compared with those described in [5]. Typically, only two or three modes are needed in the region of the uniform line sections to attain reasonable accuracy.

Simulation results for the linearly tapered microstrip discontinuity depend on: the number of uniform sections of transmission line used in the model (and corresponding number of abrupt steps); the number of modes taken on the uniform sections; the number of Legendre polynomial basis functions; the number of harmonic basis functions. Convergence results are presented which assess the sensitivity of overall results (viz. S_{11} versus frequency) to each of these parameters. Reasonable results are demonstrated for the performance of the linearly tapered microstrip discontinuity up to 15 GHz using: three to five Legendre basis functions, 140 harmonic basis functions, eight sections of uniform lines, and two modes on uniform line section where the line width is 0.25 mm at the narrow end of the taper and 0.635 mm at the wide end. It is found that the computation of S_{11} at one frequency (viz. 4 GHz) takes 25 minutes and requires about 5.3 MBytes of storage on a SUN SPARCstation model 10.

REFERENCES

[1] T. Itoh, and R. Mittra, "Spectral domain approach for calculating the dispersion characteristics of microstrip lines," *IEEE Trans. Micr. Th. Techs.*, Vol. 21, pp. 496-9, Jul. 1973.
[2] A. Wexler, "Solution of waveguide discontinuities by modal analysis," *IEEE Trans. Micr. Th. Techs.*, Vol. 15, pp. 508-17, Sep. 1967.
[3] K. Ganguly and B. Spielman, "Dispersion characteristics for arbitrarily-configured transmission media," *IEEE Trans. Micr. Th. Techs.*, Vol. 25, pp. 1138-41, Dec. 1977.
[4] K. Suryanarayana Rao and S. Kosta, "On tapered microstrip transmission lines," *Asia Electronics Union*, Vol. 7, No. 1, 1974.
[5] Q. Xu and K. Webb, "Study of modal solution procedures for microstrip step discontinuities," *IEEE Trans. Micr. Th. Techs.*, Vol. 37, No. 2, pp. 381-7, Feb. 1989.

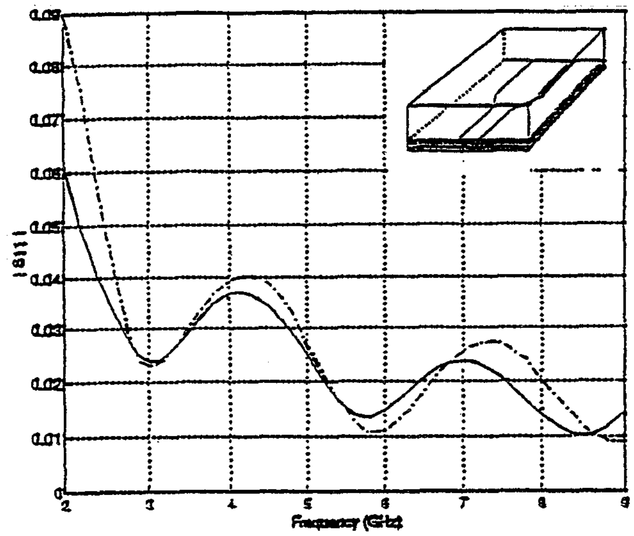


Fig. 12. Simulation results of | S_{11} | as a functions of frequency for the shown microstrip with tapered discontinuities (w_a=0.25mm, w_b=0.635mm, a=6.35mm, h=12.7mm, d=1.27mm, L=20mm, ε_r=10, harmonic terms=140, the number of basis functions=3, the number of modes=2, the number of sections=8) ——— Chen & Spielman, –·–·– by Rao & Kosta.